

## Module : Modélisation et Simulation

### Correction des Exercices TD N °3

#### Exo N° 1: Matrice de transition

La matrice de transition :

$$M = \begin{bmatrix} 1 & 0 \\ 0.5 & 0.5 \end{bmatrix} \quad \mathbf{p}^{(0)} = \left( \frac{1}{3}, \frac{2}{3} \right)$$

$$M_{21}^{(0)} = 0.5$$

$$M^{(3)} = M^{(2)} M = \begin{bmatrix} 1 & 0 \\ 0.5 & 0.5 \end{bmatrix}$$

$$\mathbf{p}^{(3)} = M^{(3)} \mathbf{p}^{(0)} =$$

#### Exo N° 2 : Vecteur de probabilité limite

Matrice de transition :

$$M = \begin{bmatrix} 0.0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.0 \\ 0.0 & 1.0 & 0 \end{bmatrix} \quad \mathbf{p}^{(0)} = \left( \frac{2}{3}, 0, \frac{1}{3} \right)$$

$$1 - M^{(3)} = M^{(2)} M = \begin{bmatrix} \frac{1}{4} & \frac{3}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

- $M_{32}^{(3)} = \frac{1}{2}$
- $M_{13}^{(2)} = 0$

$$\mathbf{p}^{(4)} = \left( P_1^{(4)}, P_2^{(4)}, P_3^{(4)} \right) = \mathbf{p}^{(0)} M^4 = \left( \frac{1}{4}, \frac{7}{12}, \frac{1}{6} \right)$$

- $P_3^{(4)} = \frac{1}{6}$

$$2 - P^\infty = P^{(0)} M^\infty = P^{(*)} M$$

$$P^{(*)} = (x, y, 1 - x - y)$$

$$P^{(*)} = P^{(*)} M$$

$$P^{(*)} = \left( \frac{2}{7}, \frac{4}{7}, \frac{1}{7} \right)$$

$$3 - M^{(*)} = \begin{bmatrix} \frac{2}{7} & \frac{4}{7} & \frac{1}{7} \\ \frac{2}{7} & \frac{4}{7} & \frac{1}{7} \\ \frac{2}{7} & \frac{4}{7} & \frac{1}{7} \end{bmatrix}$$