Solutions TD 1

Exo 1

1)

$$I_1 = \int \frac{1}{\alpha^2 + x^2} dx \text{ , we put } X = \frac{x}{\alpha}.$$

$$I_1 = \frac{1}{\alpha} \int \frac{\alpha}{X^2+1} dX = \frac{1}{\alpha^2} \operatorname{artg} X + C$$

$$I_2 = \int_1^4 \frac{e^{\sqrt{x}} + e^{2\sqrt{x}}}{\sqrt{x}} dx \text{ ; we put: } t = \sqrt{x}.$$

we get: $I_2 = 2\int_1^2 (e^t + e^{2t}) dt.$

$$= e^4 + e^2 - 2e.$$

$$I_3 = \int \frac{\cos x}{1+\sin x} dx \text{ ; we put: } u = 1+\sin x.$$

$$du = \cos x \, dx.$$

$$I_3 = \int \frac{du}{u} = \ln|u| + C = \ln(1+\sin x) + C.$$

2) $J_1 = \int \arcsin x \, dx$

$$= x \arcsin x + \sqrt{1-x^2} + C.$$

$$J_2 = \int (x^2+1) \ln x \, dx.$$

$$= \left(\frac{x^3}{3} + x\right) \ln x - \frac{x^3}{9} - x + C.$$

$$J_3 = \int e^x \cos x \, dx$$
$$= e^x \sin x - \int e^x \sin x \, dx$$
$$= \frac{1}{2} e^x (\sin x + \cos x).$$

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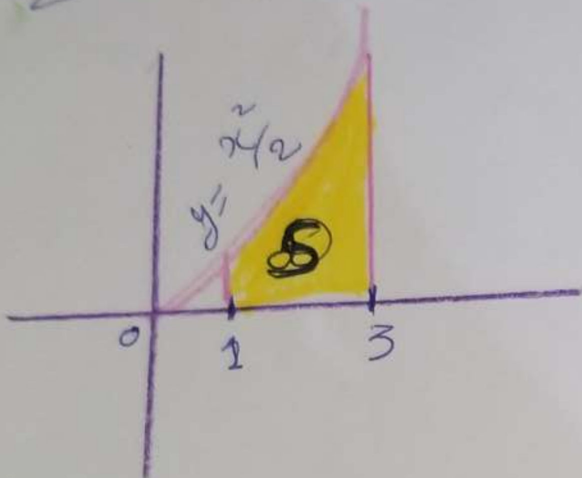

$$S = \int_1^3 \frac{x^2}{2} \, dx = \frac{13}{3}.$$

Ex 02 $\iint_D \frac{1}{(x+y)^3} \, dx\, dy =$

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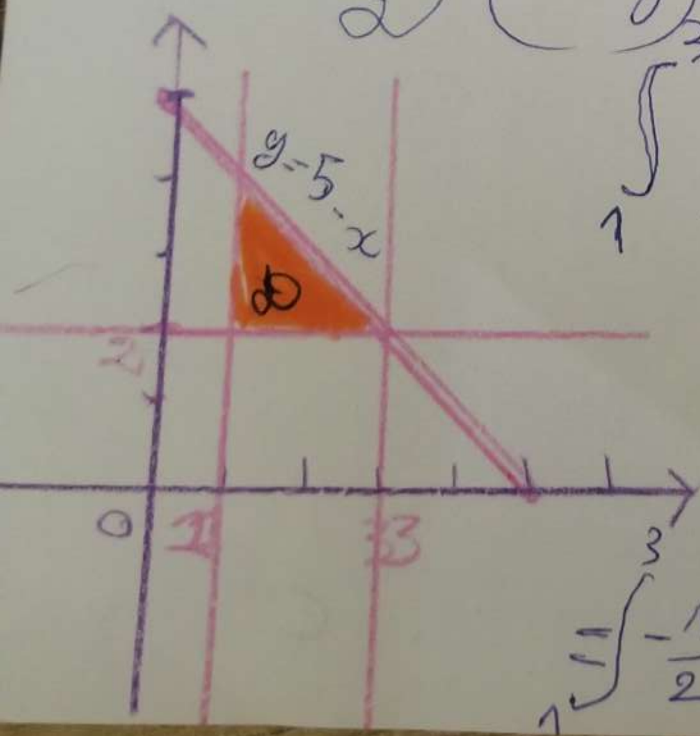
$$\int (\alpha x+\beta)^n dx = \frac{1}{\alpha}\frac{(\alpha x+\beta)^{n+1}}{n+1}$$



$$\int_1^3 \int_2^{5-x} \frac{1}{(x+y)^3} \, dy\, dx$$

$$= \int_1^3 \int_2^{5-x} (x+y)^{-3} \, dy\, dx.$$

$$= \int_1^3 -\frac{1}{2}\left[\frac{1}{(x+y)^2}\right]_2^{5-x} = \int_1^3 -\frac{1}{50} + \frac{1}{2}\frac{1}{(x+2)^2} \, dx$$

$$= \left[\frac{-1}{80} x - \frac{1}{2} \frac{1}{x-2} \right]_1^3 = .$$

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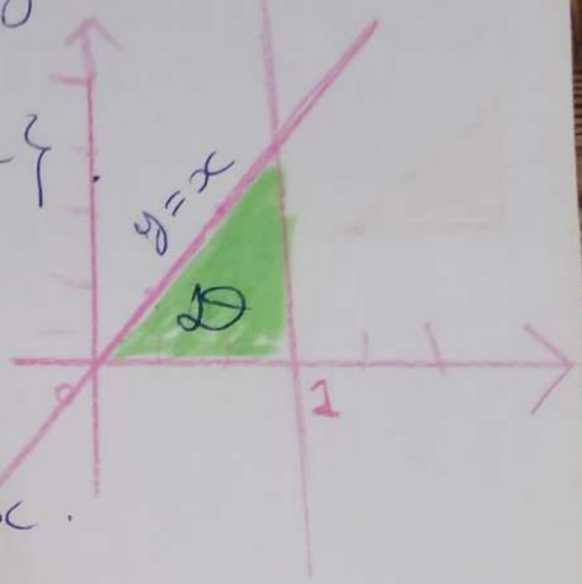
$$\iint_D \frac{1}{(1+x^2)(1+y^2)} \, dx \, dy .$$

$D = \{ 0 \le y \le x \le 1 \}$.



we have: $0 \le x \le 1$

$0 \le y \le x$.

$$\int_0^1 \int_0^x \frac{1}{(1+x^2)(1+y^2)} \, dy \, dx .$$

$$= \int_0^1 \left(\frac{1}{1+x^2} \int_0^x \frac{1}{1+y^2} \, dy \right) dx .$$

$$= \int_0^1 \frac{1}{1+x^2} \left[\operatorname{Arctg} y \right]_0^x \, dx .$$

$$= \int_0^1 \frac{1}{1+x^2} \operatorname{Arctg} x \, dx = \frac{\pi^2}{32}$$

here like $\int f' f^n = \frac{f^{n+1}}{n+1}$ with $n=1$.

<u>Remark</u>: we can put $\{ 0 \le y \le 1,\ y \le x \le 1 \}$

$$\iint_{D} x\,y\,dx\,dy$$

$xy + x + y \le 1.$

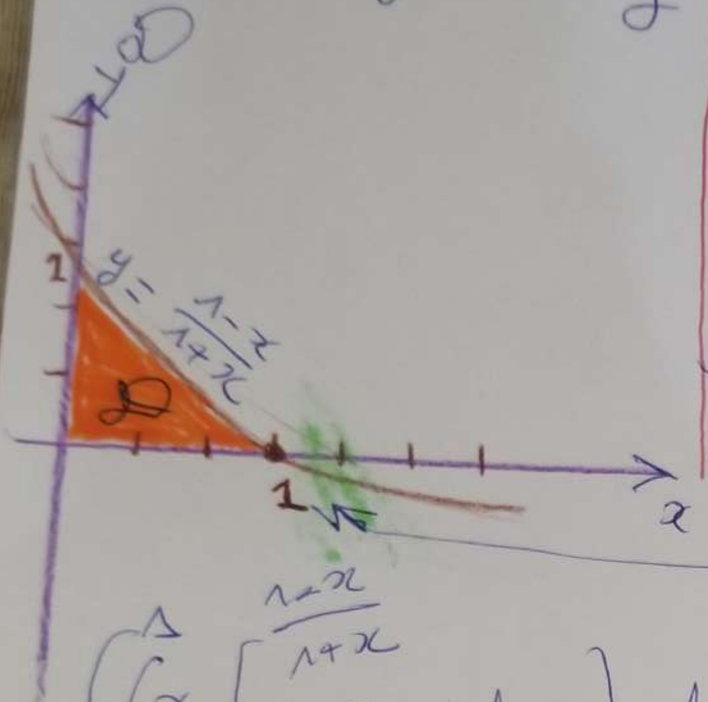
$xy + y \le 1 - x.$

$y(x+1) \le 1 - x.$

$y \le \dfrac{1-x}{1+x}.$

so: $0 \le y \le \dfrac{1-x}{1+x}$

and: $0 \le x \le 1$



$$\int_0^1 \left(x \int_0^{\frac{1-x}{1+x}} y\,dy \right) dx = \int_0^1 \frac{x^3 - 2x^2 + x}{2x^2 + 4x + 2}\,dx.$$

$x^3 - 2x + x$ | $2x^2 + 4x + 2$

$\frac{1}{2}x - 2$

$8x + 4$

we obtain.

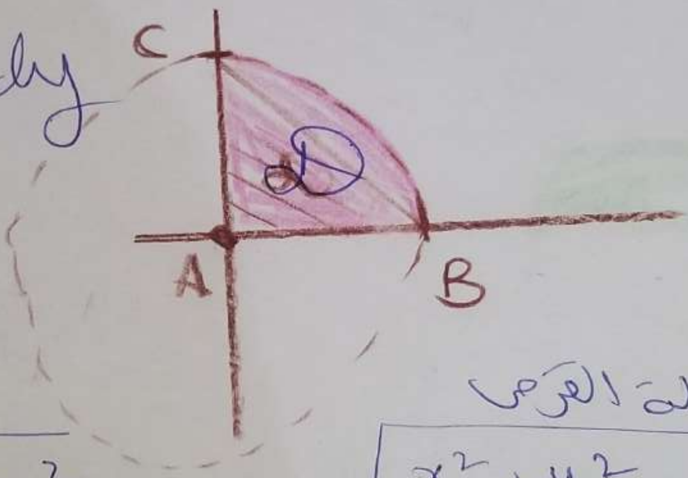
$$\iint_{D} x\,y\,dx\,dy = \int_0^1 \left(\frac{1}{2}x - 2\right) dx + \int_0^1 \frac{2(4x+2)}{2\underbrace{(x^2 + 2x + 1)}_{\Delta = 0}}\,dx = \int_0^1 \left(\frac{1}{2}x - 2\right) dx.$$

$$+\int_0^1 \left(\frac{4}{(x+1)} - \frac{2}{(x+1)^2} \right) dx.$$ (5)

because $\frac{4x+2}{(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2}$.

$$= 4 \ln|x+1| + 2 \frac{1}{(x+1)} \Big]_0^1$$

$$\iint_{\mathcal{D}} x e^y \, dx \, dy$$



here: $0 \le x \le 1$.

$0 \le y \le \sqrt{1-x^2}$

معادلة القرص

$x^2 + y^2 \le 1$

$y^2 \le 1 - x^2$.

$y \le \pm \sqrt{1-x^2}$

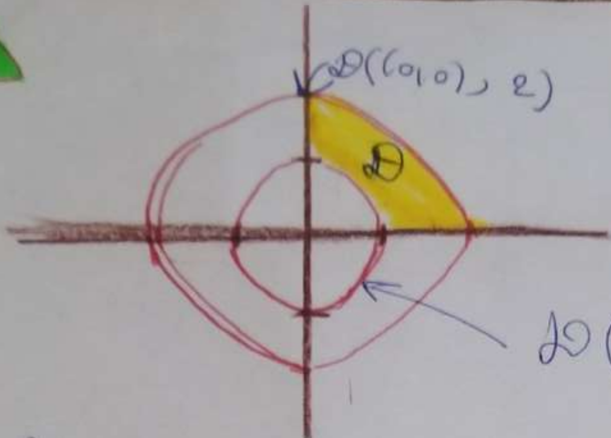
$$\to \int_0^1 x \int_0^{\sqrt{1-x^2}} e^y \, dy$$

$$= \underbrace{\int_0^1 x \, e^{\sqrt{1-x^2}} \, dx}_{I} - \int_0^1 x \, dx.$$

$I = ?$, we put $t = \sqrt{1-x^2} \to \int -t e^t dt$

Integre by parts. finally $= \frac{1}{2}$.

Exo3



here $f(x,y) = 1$

$$\iint 1\, dx\, dy = \int_0^{\frac{\pi}{2}}\int_1^2 1\, r\, dr\, d\theta = \frac{3\pi}{4}.$$

$1 \leq r \leq 2$

$0 \leq \theta \leq \frac{\pi}{2}$

Exo4.

$$\iiint_V \frac{1}{(1+x+y)^3}\, dx\, dy\, dz$$

$\cancel{x+y+z < 1}$

$\Rightarrow 0 < x < 1-y-z$

$0 < 1-y-z$

$\Rightarrow 0 < y < 1-z$

$0 < 1-z$

$\Rightarrow 0 < z < 1$

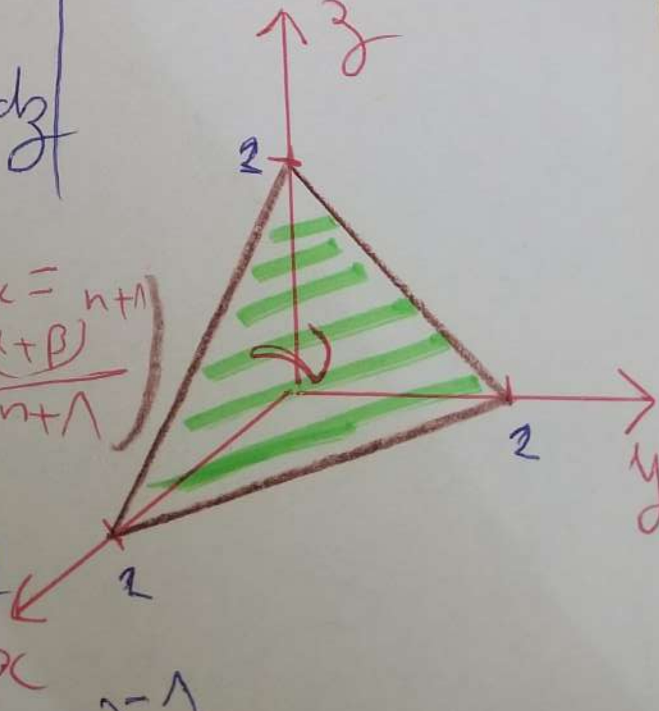
$$\int_0^1 \int_0^{1-z} \int_0^{1-y-z} \frac{1}{(1+x+y)^3}\, dx\, dy\, dz$$

$$\left(\int (\alpha x+\beta)^n dx = \frac{1}{\alpha} \frac{(\alpha x+\beta)^{n+1}}{n+1} \right)$$

$$-\frac{1}{8} + \frac{1}{2}(1+y+z)^{-2}$$

$$\int_0^{1-z} -\frac{1}{8} + \frac{1}{2}(1+y+z)^{-2} dy$$

$$= -\frac{1}{8} + \frac{1}{8} z - \frac{1}{4} + \frac{1}{2}(1+z)^{-1}$$



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$$\int_0^1 \frac{-3}{8} + \frac{1}{8}z + \frac{1}{2}(1+z)^{-1}\, dz.$$

$$= \frac{-5}{16} + \frac{1}{2}\ln 2.$$

$$\iiint_V xz \sin(xy)\, dx\, dy\, dz$$ Devoire

$0 \le z \le 2$, $1/6 \le x \le 1$, $0 \le y \le \pi$.

$$\iiint_V dx\, dy\, dz$$ $-1 \le x \le 1$, $\sqrt{1-x^2} \le y \le \sqrt{1-x^2}$

$0 \le z \le y+1$.

$$= \int_{-1}^{+1} \int_{-\sqrt{1-x^2}}^{\sqrt{1+x^2}} \int_0^{y+1} dz\, dy\, dx.$$

$$= \int_{-1}^{+1} \int_{-\sqrt{1-x^2}}^{\sqrt{1+x^2}} \left[z\right]_0^{y+1} dy\, dx.$$

$$= \int_{-1}^{+1} \int_{-\sqrt{1-x^2}}^{\sqrt{1+x^2}} (y+1)\, dy\, dx.$$

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$$= \int_{-1}^{+1} \left[\frac{y^2}{2} + y \right]_{-\sqrt{1-x^2}}^{\sqrt{1+x^2}} dx.$$

$$= \int_{-1}^{+1} 2\sqrt{1-x^2}\, dx.$$

$$= 2 \left[\frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1}(x) \right]_{-1}^{+1}$$

$$= \pi$$

Exo 6 $\int_{1}^{+\infty} \frac{1}{x^{5/2}} dx$. Devoire

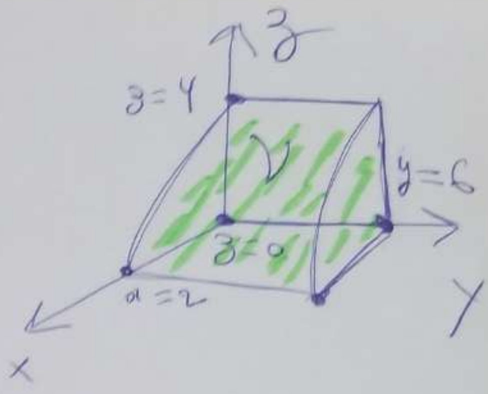
$$\int_{-\infty}^{\pi} \cos x \, dx = \lim_{b \to -\infty} \int_{b}^{\pi} \cos x \, dx.$$

$$= \lim_{b \to -\infty} (\sin \pi - \sin b) = \nexists \to \text{Div.}$$

$$\int_{-\infty}^{+\infty} e^{x} dx = \underbrace{\int_{-\infty}^{0} e^{x} dx}_{\nexists} + \underbrace{\int_{0}^{+\infty} e^{x} dx}_{\nexists}.$$

$\longrightarrow$ Div

Ex 05



$$V = \iiint_D dx\,dy\,dz$$

$$= \int_0^2 \int_0^6 \int_0^{4-x^2} dz\,dy\,dx \,.= 32 .$$

$$= \int_0^2 \int_0^6 (4-x^2)\,dy\,dx$$