ANN1 Série 3 Dr. Khelladi Samia.

Exo 2

2) $f(x) = \sin x$, $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$.

Supposons qu'on n'a pas remarqué que f est impaire

(لم ننتبه أن الدالة f فردية)

Alors :

$\left(P_1(x) = ax + b \right.$ est la m. app. au sens des M.C de f sur $\left.[-\frac{\pi}{2}, \frac{\pi}{2}] \right)$

$$\Leftrightarrow \begin{cases} \langle f(x) - P_1(x), 1 \rangle = 0 \\ \langle f(x) - P_1(x), x \rangle = 0 \end{cases}$$

On a :

$* \ \langle f(x) - P_1(x), 1 \rangle = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin x - ax - b) \cdot 1 \cdot w(x)\, dx = 0 \ / \ w(x) = 1$.

$$\Rightarrow \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x - ax - b \, dx = 0$$

$$\Rightarrow \left[-\cos x - \frac{a}{2}x^2 - bx \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 0$$

$$\Rightarrow \left(-\cos(\tfrac{\pi}{2}) - \frac{a}{2}(\tfrac{\pi}{2})^2 - b(\tfrac{\pi}{2}) \right) - \left(-\cos(-\tfrac{\pi}{2}) - \frac{a}{2}(-\tfrac{\pi}{2})^2 - b(-\tfrac{\pi}{2}) \right) = 0$$

$$\Rightarrow \left(0 - \frac{a}{2}\frac{\pi^2}{4} - b\frac{\pi}{2} \right) - \left(0 - \frac{a}{2}\frac{\pi^2}{4} + \frac{b\pi}{2} \right) = 0.$$

$$\Rightarrow -b\pi = 0 \Rightarrow \boxed{b = 0}$$

Mais si on remarque que $f(x)$ est impaire alors, on a $P_1(x)$ impaire $\Rightarrow P_1(x) = ax$ $(b = 0)$ directement sans calculs.

$* \ \langle f(x) - P_1(x), x \rangle = 0 \Leftrightarrow \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin x - ax)\, x \, dx = 0$ $\left(\begin{matrix} w(x) = 1 \\ b = 0 \end{matrix} \right)$

$$\Leftrightarrow \left[-x\cos x + \sin x - \frac{a}{3}x^3 \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 0 \Leftrightarrow \boxed{a = \frac{24}{\pi^3}}$$

et $\boxed{P_1(x) = \frac{24}{\pi^3} x}$